

Department of Electrical and Computer Engineering

ECED4601 Digital Control System

Lab 4 Design of Discrete-time Control System

Objectives:

- To learn some MATLAB commands that deals with the discrete time systems.
- To give the students the ability to program using MATLAB.
- To transform between transfer function and state space model of the system.
- To design a state feedback controller given desired pole locations.
- Implement a state feedback controller on the Feedback using the MATLAB Simulink
- To design State Feedback and Observer-Based Controllers

Overview

We will design better controllers if we have more measurements of the system behavior. In the case where we can measure all of the system states, we can place the closed-loop poles anywhere we choose within the constraint of actuator saturation also use any type of control system.

1) For a discrete-time state space model

$$x(k+1) = \begin{bmatrix} 0.13 & 0.154 & 0.328 & -0.046 \\ -0.126 & 0.085 & -0.232 & -0.319 \\ 0.128 & -0.376 & 0.783 & 0.134 \\ 0.318 & -0.081 & 0.117 & 0.044 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ -1.292 \\ 0 \\ -0.331 \end{bmatrix} u(k)$$

$$\text{with } y(k) = [-0.8440 \quad 0.497 \quad 1.488 \quad -0.547]x(k) - 0.8468u(k)$$

- a) Calculate the controllability Matrix and the observability matrix and demonstrate that the system is fully controllable and fully observable.
- b) Design a state feedback vector K which places the closed loop poles of the system at, $-0.5 \pm j0.5, \pm 0.1$

2) Consider the discrete-time state space model given

$$x(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.91 & -0.036 & 0.91 & 0.036 \\ 0 & 0 & 0 & 1 \\ 0.91 & 0.036 & -0.091 & -0.036 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = x_1(k) \quad , \quad D = 0$$

With sampling interval $T=0.1$ second. If one wants to place all the closed-loop poles at 0.9

Determine

- a) The state feedback vector K
- b) The poles of the closed-loop system

3) Consider the fourth order discrete time system

$$x(k+1) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -4.1504 & -1.1638 \\ 0 & 0 & 0 & 1 \\ 0.91 & 0.036 & -0.091 & -0.036 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 2 \quad 3 \quad 4] x(k) \quad , \quad D = 0$$

- a) Design the observer gain matrix K_e which places the closed loop poles of the system at, $-0.3 \pm j0.4$, ± 0.3

4) Consider the discrete-time state space equation

$$x(k+1) = \begin{bmatrix} 0.4725 & 0.2376 & 0.0589 & 0.1971 \\ 0.1451 & 0.5669 & 0.2311 & 0.0439 \\ 0.0932 & 0.119 & 0.5752 & 0.2319 \\ 0.2628 & 0.0757 & 0.1406 & 0.4465 \end{bmatrix} x(k) + \begin{bmatrix} 0.5711 \\ -0.3999 \\ 0.6899 \\ 0.8156 \end{bmatrix} u(k)$$

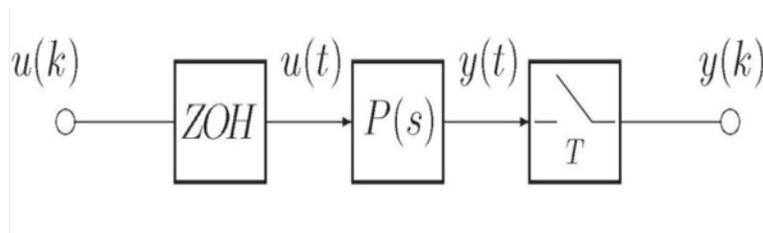
$$Q = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.4 \end{bmatrix} \quad R = 1$$

Determine

- The Riccati Matrix P
- The state feedback matrix K
- The closed-loop Poles

5) In this problem you will derive a state space representation and state feedback controller.

$$P(s) = \frac{1000}{(s^2 + 12s + 100)(s + 100)}$$



The standard sampled data configuration.

With a sample time of $T=0.0002$ seconds the corresponding discrete-time transfer function from $u(k)$ to $y(k)$ is:

$$P(z) = \frac{10^{-9}(1.325z^2 + 5.274z + 1.3111)}{z^3 - 2.9778z^2 + 2.9556z - 0.9778}$$

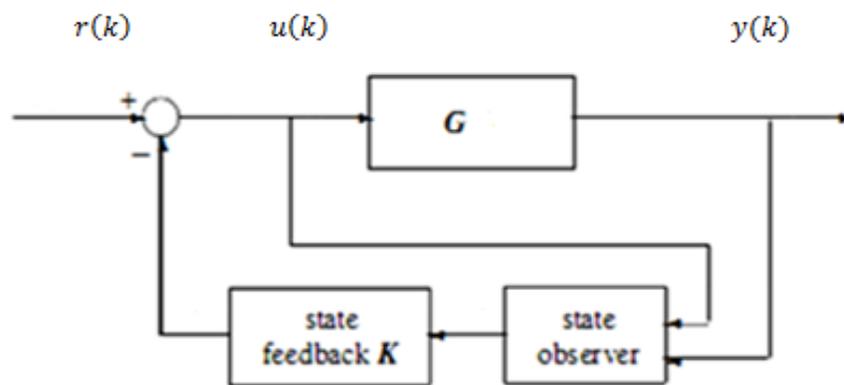
Determine

- a) To represent the discrete-time dynamics $P(z)$ in state-space format, how many states are required in the state vector x ?
- b) The matrices G , H , C , and D to represent this discrete-time state space system.

$$x(k + 1) = Gx(k) + Hu(k)$$

$$y(k) = Cx(k) + Du(k)$$

6) State Feedback and Observer-Based Controllers. The state feedback control strategy with observers can be implemented as shown in Figure Below



Feedback control with a state observer

Consider the following plant in state space form:

$$x(k+1) = \begin{bmatrix} 0.9048 & 0.2101 & 0.0206 & 0.0499 & 0.3872 \\ 0 & 0.7788 & 0.0902 & 0.2358 & 2.3987 \\ 0 & 0 & 0.0008 & 0.0035 & 0.4946 \\ 0 & 0 & 0 & 0 & 0.0289 \\ 0 & 0 & 0 & 0 & 0.0067 \end{bmatrix} x(k) + \begin{bmatrix} 2.0896 \\ 24.0652 \\ 52.4959 \\ 8.9223 \\ 2.9798 \end{bmatrix} u(k)$$

$$y(k) = [1 \ 0 \ 0 \ 0 \ 0], \quad D = 0$$

If the observer gain vector is selected as: $Ke = [-8.3 \ 979.24 \ -19367.61 \ 4293.85 \ 0]^T$

and the state feedback gain vector is selected as :

$$K = [1.3563 \ 0.0858 \ 0.0085 \ 0.0206 \ 0.1623]$$

For Design the observer-based controller, using the MATLAB statements:

Determine

- a) Design the state space observer-based regulator depends on the observer gain vector Ke and the state feedback gain vector K

Guidelines for writing the Lab Report

The report has to include:

- ❖ Title page - Including: course number, lab title, student names, student IDs, Date the report was submitted.
- ❖ Abstract - A summary of the contents of the lab report.
- ❖ Procedure of methods or approach to the design or/and conduct of the experiments
- ❖ Diagrams - Include all SIMULINK blocks used in the lab if any.
- ❖ Plots - All plots of system response should be included in your report, including title, labels with unit, legends, etc. Note that Diagrams and Plots must be computer generated. Hand-drawn plots will not be accepted. All diagrams and plots must be labeled. The labels and annotations can be done by hand if appropriate but have to be clear.
- ❖ Code - Include any code used in the lab. The code must be commented properly. It is good practice to put your code in separate m-files.
- ❖ Discussion and answer to questions if any.
- ❖ Conclusion or any other relevant ideas.

MATLAB command prompt:

- c2d Continuous state-space to discrete state-space conversion
- residue Partial fraction expansion conversion
- ss2tf State-space to transfer function conversion
- ss2zp State-space to zero-pole-gain conversion
- tf2ss Transfer function to state-space conversion
- tf2zp Transfer function to zero-pole-gain conversion
- zp2ss Zero-pole-gain to state-space conversion
- zp2tf Zero-pole-gain to transfer function conversion

The basic design and analysis functions a summarized below:

- rlocus Evans root-locus plot

- rlocfind Locating the system gain from pole locations indicated by the mouse
- nyquist Nyquist frequency response plot
- bode Log-magnitude and phase vs. frequency response plot
- place Pole placement state feedback gain determination (A-BK)
- lqr Linear-quadratic regulator determination for optimal control
- lqe Linear-quadratic estimator determination for optimal control
- step Unit step time response plot
- lsim Time response simulation for generic input functions

Examples

Given a continuous-time state-space model

$$\text{sys} = \text{ss}(A,B,C,D)$$

With seven outputs and four inputs, suppose you have designed:

- A state-feedback controller gain K using inputs 1, 2, and 4 of the plant as control inputs
- A state estimator with gain L using outputs 4, 7, and 1 of the plant as sensors, and input 3 of the plant as an additional known input

You can then connect the controller and estimator and form the complete regulation system by

$$\text{controls} = [1,2,4];$$

$$\text{sensors} = [4,7,1];$$

$$\text{known} = [3];$$

$$\text{regulator} = \text{reg}(\text{sys},K,L,\text{sensors},\text{known},\text{controls})$$

Pole placement design for single-input systems

$$k = \text{acker}(A,B,p)$$

use acker for estimator gain

$$L = \text{acker}(A',C',p)$$

Feedback connection of two LTI models

$$\text{sys} = \text{feedback}(\text{sys1},\text{sys2})$$

$$\text{rsys} = \text{reg}(\text{sys},K,L)$$

$$\text{rsys} = \text{reg}(\text{sys},K,L,\text{sensors},\text{known},\text{controls})$$

reg

Form regulator given state-feedback and observer gains