

ECED 4601 Digital Control Systems

Midterm

Date: October 27 2016

Room: Sexton Gym

Time: 2:30PM-4:30PM

Student name	
ID	

1) Obtain the inverse z transform of $X(z) = \frac{z^{-3}}{(1-z^{-1})(1-0.2z^{-1})}$

a) Using inversion integral method

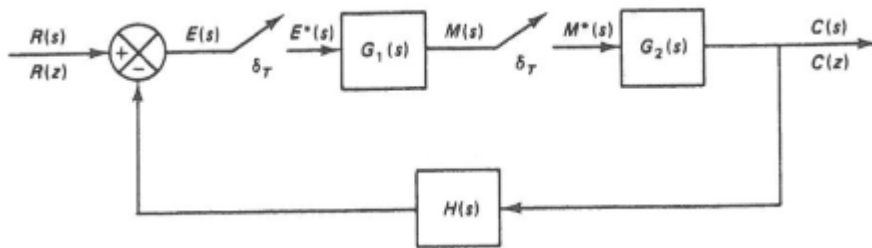
b) Using partial-fraction expansion method

2) Find the solution of the following difference equation in a closed form.

$$x(k+2) = x(k+1) + x(k)$$

Where $x(0) = 0, x(1) = 1$

3) Consider the system shown in figure. Obtain the close-loop pulse transfer function $\frac{C(z)}{R(z)}$. Also obtain the expression for $C(s)$



4) Consider the digital filter defined by

$$G(z) = \frac{2 + 2.2z^{-1} + 0.2z^{-2}}{1 + 0.4z^{-1} - 0.12z^{-2}}$$

Realize the digital filter in

a) The parallel scheme,

b) The ladder scheme.

5) Consider the system described by

$$y(k) - 0.6y(k-1) - 0.81y(k-2) + 0.67y(k-3) - 0.12y(k-4) = x(k)$$

Determine the stability of the system:

Laplace Transform Pairs for Bilateral

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\text{Re}(s) > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\text{Re}(s) < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\text{Re}(s) > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\text{Re}(s) < 0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\text{Re}(s) > -a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\text{Re}(s) < -a$
8	$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\text{Re}(s) > -a$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\text{Re}(s) < -a$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos w_0 t]u(t)$	$\frac{s}{s^2 + w_0^2}$	$\text{Re}(s) > 0$
12	$[\sin w_0 t]u(t)$	$\frac{w_0}{s^2 + w_0^2}$	$\text{Re}(s) > 0$
13	$[e^{-at} \cos w_0 t]u(t)$	$\frac{s+a}{(s+a)^2 + w_0^2}$	$\text{Re}(s) > -a$
14	$[e^{-at} \sin w_0 t]u(t)$	$\frac{w_0}{(s+a)^2 + w_0^2}$	$\text{Re}(s) > -a$
15	$u_n(t) = \frac{d^n \delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = u(t) * \dots * u(t)$	$\frac{1}{s^n}$	$\text{Re}(s) > 0$

Laplace Transform Property for Bilateral

Property	Signal	Laplace Transform	ROC
	$x(t), x_1(t), x_2(t)$	$X(s), X_1(s), X_2(s)$	R, R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(S) + bX_2(S)$	$R_1 \cap R_2$
Time Shifting	$x(t - t_0)$	$e^{-st_0} X(S)$	R
Shift in the s-Domain	$e^{s_0 t} x(t)$	$X(s - s_0)$	R
Time Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	Scaled ROC
Conjugation	$x^*(t)$	$X^*(S^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the time-Domain	$\frac{dx(t)}{dt}$	$sX(S)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{dX(S)}{dS}$	R
Integration in the Time Domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{S} X(S)$	At least $R \cap \{\text{Re}(s) > 0\}$
Initial value theorem	$x(0^+) = \lim_{s \rightarrow \infty} sX(S)$		
Final value theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(S)$		

Laplace Transform Property for Unilateral

Property	Signal	Laplace Transform
	$x(t), x_1(t), x_2(t)$	$\chi(s), \chi_1(s), \chi_2(s)$
Linearity	$ax_1(t) + bx_2(t)$	$a\chi_1(S) + b\chi_2(S)$
Shift in the s-Domain	$e^{s_0 t} x(t)$	$\chi(s - s_0)$
Time Scaling	$x(at)$	$\frac{1}{ a } \chi\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$\chi^*(S)$
Convolution	$x_1(t) * x_2(t)$	$\chi_1(s)\chi_2(s)$
Differentiation in the time-Domain	$\frac{dx(t)}{dt}$	$S\chi(S) - x(0^-)$
	$\frac{d^2 x(t)}{dt^2}$	$S^2 \chi(S) - sx(0^-) - x'(0^-)$
Differentiation in the s-Domain	$-tx(t)$	$\frac{d\chi(S)}{dS}$
Integration in the Time Domain	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{S} \chi(S)$
Initial value theorem	$x(0^+) = \lim_{s \rightarrow \infty} S\chi(S)$	
Final value theorem	$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s\chi(S)$	

Table of z transforms 1

	X(s)	x(t)	x(kT) or x(k)	X(z)
1.	—	—	Kronecker delta $\delta_0(k)$ 1, $k = 0$ 0, $k \neq 0$	1
2.	—	—	$\delta_0(n - k)$ 1, $n = k$ 0, $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	1(t)	1(k)	$\frac{1}{1 - z^{-1}}$
4.	$\frac{1}{s + a}$	e^{-at}	e^{-akT}	$\frac{1}{1 - e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1 - z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1 + z^{-1})}{(1 - z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$
8.	$\frac{a}{s(s + a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1 - e^{-aT})z^{-1}}{(1 - z^{-1})(1 - e^{-aT}z^{-1})}$
9.	$\frac{b - a}{(s + a)(s + b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1 - e^{-aT}z^{-1})(1 - e^{-bT}z^{-1})}$
10.	$\frac{1}{(s + a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s + a)^2}$	$(1 - at)e^{-at}$	$(1 - akT)e^{-akT}$	$\frac{1 - (1 + aT)e^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$

Table of z transforms 2

	X(s)	x(t)	x(kT) or x(k)	X(z)
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT} (1 + e^{-aT} z^{-1}) z^{-1}}{(1 - e^{-aT} z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT - 1 + e^{-aT}) + (1 - e^{-aT} - aTe^{-aT})z^{-1}]z^{-1}}{(1 - z^{-1})^2(1 - e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1 - z^{-1} \cos \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1 - e^{-aT} z^{-1} \cos \omega T}{1 - 2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.			a^k	$\frac{1}{1 - az^{-1}}$
19.			a^{k-1} $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1 - az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1 - az^{-1})^2}$
21.			$k^2 a^{k-1}$	$\frac{z^{-1}(1 + az^{-1})}{(1 - az^{-1})^3}$
22.			$k^3 a^{k-1}$	$\frac{z^{-1}(1 + 4az^{-1} + a^2 z^{-2})}{(1 - az^{-1})^4}$
23.			$k^4 a^{k-1}$	$\frac{z^{-1}(1 + 11az^{-1} + 11a^2 z^{-2} + a^3 z^{-3})}{(1 - az^{-1})^5}$
24.			$a^k \cos k\pi$	$\frac{1}{1 + az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1 - z^{-1})^3}$
26.			$\frac{k(k-1)\dots(k-m+2)}{(m-1)!}$	$\frac{z^{-m+1}}{(1 - z^{-1})^m}$
27.			$\frac{k(k-1)}{2!} a^{k-2}$	$\frac{z^{-2}}{(1 - az^{-1})^3}$
28.			$\frac{k(k-1)\dots(k-m+2)}{(m-1)!} a^{k-m+1}$	$\frac{z^{-m+1}}{(1 - az^{-1})^m}$

Property table:

	$x(t)$ or $x(k)$	$\mathcal{Z}[x(t)]$ or $\mathcal{Z}[x(k)]$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t) + bx_2(t)$	$aX_1(z) + bX_2(z)$
3.	$x(t + T)$ or $x(k + 1)$	$zX(z) - zx(0)$
4.	$x(t + 2T)$	$z^2 X(z) - z^2 x(0) - zx(T)$
5.	$x(k + 2)$	$z^2 X(z) - z^2 x(0) - zx(1)$
6.	$x(t + kT)$	$z^k X(z) - z^k x(0) - z^{k-1} x(T) - \dots - zx(kT - T)$
7.	$x(t - kT)$	$z^{-k} X(z)$
8.	$x(n + k)$	$z^k X(z) - z^k x(0) - z^{k-1} x(1) - \dots - zx(k - 1)$
9.	$x(n - k)$	$z^{-k} X(z)$
10.	$tx(t)$	$-Tz \frac{d}{dz} X(z)$
11.	$kx(k)$	$-z \frac{d}{dz} X(z)$
12.	$e^{-at} x(t)$	$X(ze^{aT})$
13.	$e^{-ak} x(k)$	$X(ze^{aT})$
14.	$a^k x(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^k x(k)$	$-z \frac{d}{dz} X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z \rightarrow \infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z \rightarrow 1} [(1 - z^{-1})X(z)]$ if $(1 - z^{-1})X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k) = x(k) - x(k - 1)$	$(1 - z^{-1})X(z)$
19.	$\Delta x(k) = x(k + 1) - x(k)$	$(z - 1)X(z) - zx(0)$
20.	$\sum_{k=0}^n x(k)$	$\frac{1}{1 - z^{-1}} X(z)$
21.	$\frac{\partial}{\partial a} x(t, a)$	$\frac{\partial}{\partial a} X(z, a)$
22.	$k^m x(k)$	$\left(-z \frac{d}{dz}\right)^m X(z)$
23.	$\sum_{k=0}^n x(kT)y(nT - kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty} x(k)$	$X(1)$

Table: z transform of $x(k+m)$ and $x(k-m)$

Discrete function	z Transform
$x(k+4)$	$z^4 X(z) - z^4 x(0) - z^3 x(1) - z^2 x(2) - zx(3)$
$x(k+3)$	$z^3 X(z) - z^3 x(0) - z^2 x(1) - zx(2)$
$x(k+2)$	$z^2 X(z) - z^2 x(0) - zx(1)$
$x(k+1)$	$zX(z) - zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1} X(z)$
$x(k-2)$	$z^{-2} X(z)$
$x(k-3)$	$z^{-3} X(z)$
$x(k-4)$	$z^{-4} X(z)$